<u>Cooperative Research and Development Agreement Final Report</u>

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Executive Summary of CRADA Work:

Cooperative Research and Development Agreement (CRADA) between Princeton Plasma Physics Laboratory (PPPL) and TAE Technologies (TAE) focused on numerical study of the effects of neutral beam injection (NBI) on stability properties of the advanced field-reversed configuration (FRC), which utilizes the NBI and electrode end biasing for the plasma sustainment and rotation control. Studies involved the use of the PPPL developed 3D nonlinear kinetic HYM code for linear and nonlinear stability studies and a cross-code verification between the HYM code and TAE's internally developed 3D hybrid Particle-in-Cell code, FPIC. Simulations demonstrate nonlinear saturation of the unstable modes and redistribution of the beam ions, which is consistent with experimental observations. A unique aspect of this work is that it provided insights into stability properties of compressional Alfven type of modes in a beam driven FRC. This type of eigenmodes, potentially unstable in the kinetic FRC-beam system, have been overlooked by previous studies. Presence of finite toroidal field is shown to have a nonlinear stabilizing effect on these modes. The work also predicts favorable scaling of stability of the compressional modes in FRC with increased beam power and device size, which is important for future design of the FRC experiments.

1. BACKGROUND

It has been found experimentally that NBI alone has a significant positive effect on the FRC plasma, increasing the plasma lifetime by a factor ~5-6, while electrode biasing alone has a more modest positive effect on the FRC plasma, increasing the plasma lifetime by a factor ~1.5-2. But the combination of NBI and electrode biasing together has a dramatic, synergistic interaction, increasing the plasma lifetime by a factor greater than 10, and making a significant increase in the electron temperature. It is crucially important to understand the physics of this interaction, both to achieve the goals of the C-2W program, and to be sure that we can benefit from the same physics in our planned next-step machine, Copernicus.

Experimental exploration of the parameter space associated with NBI and electrode biasing is difficult, because some of the determining parameters cannot be changed without engineering modifications, and some of the externally set parameters cannot be directly controlled. Such parameter scan can be done in 3D numerical simulations including the effects of neutral beam injection and end biasing on FRC global stability.

Neutral beam injection into FRC plasmas can have a stabilizing or destabilizing effect on global MHD modes, depending on the toroidal mode number *n*, the mode polarization, and the beam parameters. Important beam parameters include the normalized velocity, v_0/v_A, the average toroidal rotation frequency, Ω, the radial bounce frequency, ω_R, and the axial bounce frequency, ω_Z. In the MHD-like regimes with $S^* = R_s/d_i >> 1$ (where R_s and d_i are the FRC separatrix radius and colissionless ion skin depth) condition $\omega_R >> \Omega \sim \omega_z$ is satisfied. A generalized energy principle then predicts strong interaction between the beam ions and the MHD mode for a resonance $\omega_Z = |n\Omega - \omega|$, that can be stabilizing, if $n\Omega > \omega_z$. Most of previous theoretical studies concentrated on this large S* regime and assumed incompressible modes. However, experimental devices operate in the small S*, kinetic regime (S* < 10), when the radial orbit frequency is comparable to toroidal rotation frequency, $\omega_R \sim \Omega >> \omega_z$., Therefore, the radial resonances are more important for their stability, and as has been demonstrated by the simulations described below, the incompressibility assumption is not valid.

2. STATEMENT OF OBJECTIVES

The following are the technical objectives set out in the CRADA for FY 2020:

1. Identify suitable C-2W cases to model and develop corresponding equilibrium solutions for HYM code.
2. Perform beam parameter scan using 3D hybrid simulations with the HYM code to find the optimal operation parameters for stability.
3. Perform 3D nonlinear simulations to study the effects of end biasing on FRC global stability.
4. Compare simulation results with experiments.
5. Perform cross-code verification between the HYM code and TAE's code, FPIC.

3. TECHNICAL DISCUSSION OF WORK PERFORMED

3.1. Equilibrium calculations

For C-2W, an FRC equilibrium with fast ions is calculated self-consistently using in-house equilibrium solver iteratively coupled to the Monte-Carlo code. Several typical cases were chosen based on experimental measurements such as Thomson scattering and magnetics data. These equilibria were reproduced by the HYM code, where a new, more general, analytical form for the beam ion distribution function was implemented: $F_0(\varepsilon,p_\varphi)= F_1(\varepsilon)F_3(p_\varphi)$ (Fig.1a). Both full 2D and 1D (radial) equilibria were calculated to be used as initial conditions for 3D and reduced 2D (R,φ) simulations.

Using several C-2W equilibrium cases, the HYM code was configured for Advanced FRCs using its numerical capabilities for i) linear delta-f PIC energetic ions with fluid thermal species, ii) non-linear full-f PIC energetic ions with fluid thermal species, iii) the addition of beam ion sources and sinks in the full-f version to study their effect on stability, iv) linear delta-f PIC for all ions (beam and thermal), and v) nonlinear full-f PIC for both beam and thermal ions. These equilibrium and code configurations have been used to study the experimentally observed the n=2 instability, its parameter dependances and various stabilization mechanisms.

3.2. The n=2 compressional mode

Linearized (delta-f) and fully nonlinear (full-f) 3D stability calculations have been performed using the HYM code, and results were compared with 2D (uniform in axial direction) calculations and approximate analytic calculations in order to understand the physical mechanism of the instability and verify the 3D simulations. Linearized simulations have shown that low-frequency Alfven modes of compressional polarization can be excited by the beam ions, and their growth rates depend strongly on the beam ion distribution function and the FRC kinetic parameter, S*. There was no significant difference in the properties of these modes in a simplified case of 1D (radial) equilibrium. For the C-2W parameters, the n=2 was the most unstable mode, its growth rate reduced for low-S* equilibria. The fully nonlinear simulations have demonstrated that this instability saturates nonlinearly at low amplitude due to redistribution of the beam ions. Weak instability and saturation of the n=2 mode can explain the experimentally observed low-frequency n=2 perturbations. Simulations were performed using two different options of the HYM code: the MHD description or kinetic (PIC) description of the thermal ions; in the second case, the thermal and beam ions were implemented as two different species in the HYM code. The saturation of the n=2 compressional mode has been shown in both cases, but the saturation amplitudes were smaller in simulations with kinetic thermal ions.

3.3. Parameter scan

Both analytic predictions and numerical simulations have shown that for TAE experimental parameters the n=2 compressional mode (of fast magnetosonic type) is the most unstable mode. This is consistent with the experimental observations of the low-frequency n=2 perturbations, which are seen at low amplitudes. In accordance with the project objective, parameter scans were performed to study the dependence of the stability properties of the FRC with respect to relative beam density, n_b/n_0, and the FRC kinetic parameter, S*. Linear delta-f simulations have shown that the n=2 compressional mode remains the most unstable mode for various values of n_b/n_0=0.01-0.08, and the instability growth rate scales very weakly with the beam ion density $\gamma \sim n_b^{1/3} - n_b^{1/4}$ (Fig.1b). Beam density in simulations was reduced while keeping approximately the same total current, and the separatrix radius (by increasing thermal pressure). Predicted weak scaling of instability with n_b/n_e might be important for future FRC experiments with increased beam power.

Linearized simulations were also performed for FRC equilibria with different values of S* in the range S*= 3 - 5.5. It was shown that the instability growth rate reduces sharply for smaller values of S*. The dependence on S* is related to scaling of the ratio of the average beam ion orbit frequencies: $\omega_R/\Omega \sim \omega_{ci}R/v_b \sim S^*$, where Ω is the average toroidal rotation frequency, and ω_R is the radial bounce frequency. Strong n=2 instability corresponds to $\omega_R/\Omega \geq 2$, whereas condition $\omega_R/\Omega < 2$ implies weaker resonant type instability. For example, the growth rate for n_b/n_0=0.12, S*=3.6 and ω_R/Ω= 1.56 was reduced by a factor of three to $\gamma/\omega_{ci}\sim 0.03$ compared to an equilibrium case with larger S*=5.2, n_b/n_0=0.08 and ω_R/Ω= 2.

3.4. Nonlinear simulations

In addition to the linearized delta-f simulations, nonlinear full-f simulations were performed for different values of beam density and S* parameter. It was discovered that the instability can saturate nonlinearly when the growth rate is relatively small. In the beam ion density scan, the threshold for the n=2 compressional mode saturation was about n_b/n_0=0.02 for larger values of S*~5-6, whereas for S*=3.6 the instability saturated nonlinearly at the relatively small amplitude for the experimentally relevant value of n_b/n_0=0.12 (Fig.1c).

In the smaller S* case, nonlinear simulations were performed using two different models of thermal ions, MHD or kinetic (PIC). Contrary to expectations, the linear growth rate of the n=2 instability was not significantly affected by the model used for the thermal plasma. In both cases the saturation of the instability was observed with somewhat smaller amplitude obtained using the kinetic thermal ions model (Fig.1c,d). At saturation, significant changes were observed in the radial beam ion current profile (Fig.1e), the peak of the beam current density shifting to larger radius. Similar changes in the beam current profile were observed in all cases with nonlinear stabilization. Saturation of the instability is, therefore, caused by the changes in the beam ion distribution due to interaction of the NBI ions with the excited modes. The nonlinear full-f simulations of the growth and saturation of the n=2 instability, show how the n=2 perturbation can re-distribute the energetic particles in FRC, moving them from FRC-confined orbits to the mirror-confined orbits with relatively small loss rates related to the non-adiabaticity of mirror orbits.

3.5. Adding beam ions sources and sinks

New option of beam ions sources and sinks was implemented in the HYM code in order to more realistically model the beam ion injection, and to study its effect on the stability. Beam ion source was modelled as an injection of particles at a constant rate with the same distribution function as the initial beam ion distribution. Sinks were modelled by randomly removing particles at a constant rate. Both particle injection and particle losess were performed with the same time scale T_{loss}, corresponding to a steady state injection. Full-f nonlinear simulations were performed with added sinks and sources for the injection rates varied between $T_{loss} \approx 1/\gamma$ and $T_{loss} \approx 10/\gamma$, where $\tau=1/\gamma$ is the linear instability growth time. The nonlinear simulations for S*=5.2 and $n_b/n_0=0.08$ have shown the growth of the n=2 instability for all cases. Significant linear growth rate reduction was observed only for the smallest value of T_{loss}, indicating the fluid nature of this instability (‘cold beam’ type of instability) and not of the resonant (kinetic) type for these simulation parameters.

3.6. Effects of finite toroidal field

One of the objectives of the simulation study was to understand the interactions between neutral beam and applied end-biasing effects on the existing experiment C-2W. In C-2W the best performance and steady state are achieved by a combination of neutral beam (NBI) injection and electrode biasing – used to control plasma rotation / stability. The end biasing can lead to twisting of the FRC poloidal field due to plasma differential rotation, and generation of a finite toroidal field, oppositely directed at the FRC ends. The 2D and 3D simulations including finite toroidal field were performed to study the effects of finite toroidal field on stability. Linear delta-f simulations demonstrated that finite toroidal field has a weak effect on the instability growth rates. However, the nonlinear simulations discovered that toroidal field has a nonlinear stabilizing effect on the n=2 instability, even in simulations with strong linear drive. Saturation of instabilities was observed in both 2D and 3D simulations with S*=5.2 and $n_b/n_0=0.08$ when toroidal field was sufficiently large $B_\varphi/B_{ext} \geq 0.25$. Finite toroidal field and smaller perturbation amplitudes also lead to a significant reduction of the NBI losses due to the n=2 perturbation. The nonlinear stabilizing effect of finite toroidal field can, at least partially, explain the benefits of the application of the end-biasing during the NBI on C-2W.

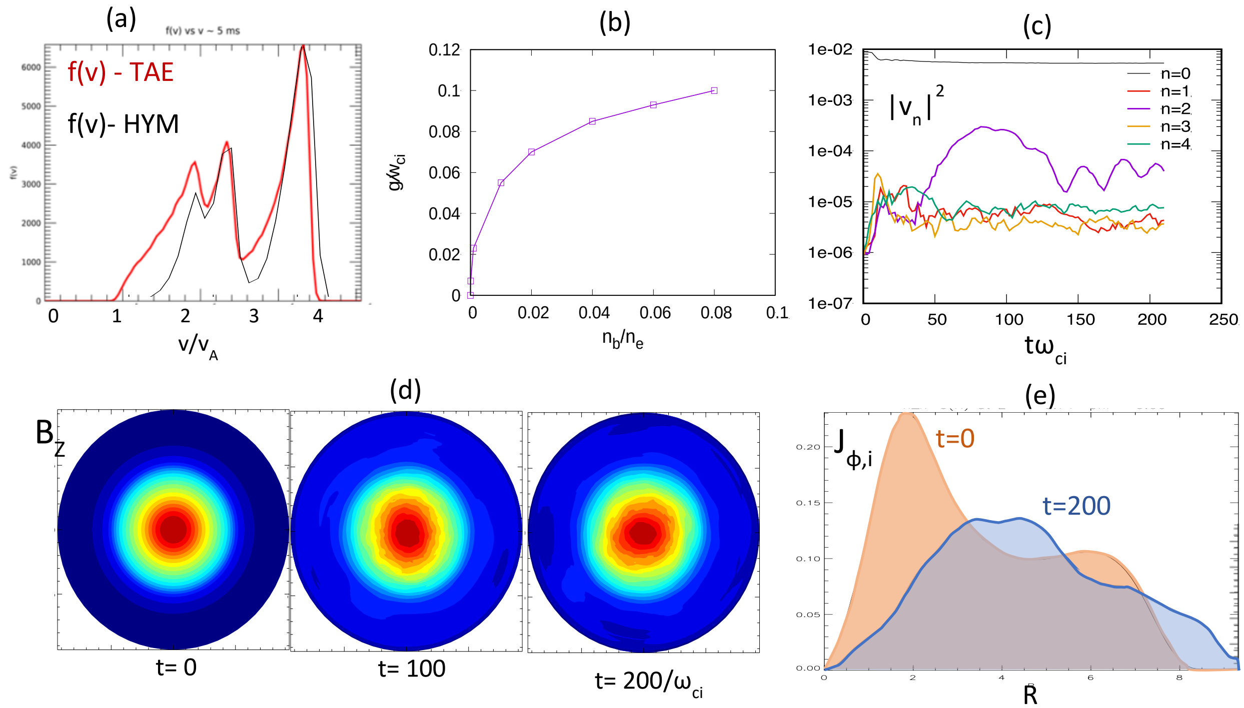

FIG 1. a) Beam ion energy distribution from equilibrium fit (red) and HYM (black) ;(b) growth rate of n=2 mode vs normalized beam ion density; Nonlinear simulations of FRC for S=3.6 with kinetic thermal and beam ions: (c) time evolution of n=0-4 Fourier harmonics of perturbed velocity, (d) contour plots of axial magnetic field, (e) change in the radial profile of the ion current.*

3.7. Analytical theory for beam-driven compressional modes

In addition to the simulation study, an analytic dispersion relation for 1D cylindrical field reversed configuration has been derived [publication to be submitted]. For a specific form of poloidal field profile, the fast ion orbits can be derived analytically, and the expression for the perturbed distribution function can be integrated along the particle trajectory. Assuming that $\bar{R} \gg \lambda > \Delta R$, where $\bar{R}$ and ΔR are the averaged beam ion orbit radius and the orbit width, and λ is the radial mode size, the dispersion relation is

$$\omega^2 = k^2 v_A^2 \left(1 - \alpha \frac{\bar{\Omega}}{(1-\bar{\Omega}^2)}\right), \quad \text{where } \bar{\Omega} = \frac{n\Omega - \omega}{\omega_R}, \quad \text{and } \alpha = \frac{\omega_{ci}}{k_\phi v_A} \frac{J_b}{n_0 v_A} \lesssim 1$$

For instability, conditions: $1 > |\bar{\Omega}|$ (that is $|n\Omega - \omega| \approx |n\Omega| < \omega_R$) and $\alpha > (1-\bar{\Omega}^2)/\bar{\Omega}$ need to be satisfied. Although this is reactive, not a resonant type of instability, the instability condition can be more easily satisfied when $\bar{\Omega} \sim 1$. For TAE parameters ($\Omega/\omega_R \approx 1/2$) it implies that the $n = 1$ will be stable because $(1-\bar{\Omega}^2)/\bar{\Omega} \approx 3/2 > \alpha$, while higher n modes will be stable for $n > \omega_R/\Omega \sim 2$. This is in agreement with simulation results showing the $n = 2$ as the strongest instability for larger S*, and reduced growth rates for smaller S*. Dispersion relation also predicts stabilization of low n compressional modes in configurations with large S*>>1.

3.8. Cross-code verification

Nonlinear saturation of instabilities in HYM simulations using kinetic thermal ions model agrees with nonlinear simulations results obtained for similar C-2W equilibria by TAE's own code, FPIC. Nonlinear relaxation of the beam current profiles by both codes also shows the localized beam current reversal at the FRC ends, which is related to the changes of the beam ion distribution in energy and toroidal angular momentum.

4. POTENTIAL APPLICATIONS, TECHNOLOGY TRANSFER

The completed work has identified key physical parameters important for stability of the FRC-beam ion system, namely the ratio ω_R/Ω which is related to S* and v_b/v_A. The analysis of the nonlinearly saturated states suggests more stable initial beam distribution and profiles, potentially leading to new operating scenarios. Experimental and engineering impacts include the opportunity to experimentally validate theoretical predictions, and to potentially achieve higher performance plasmas.

All of the above impacts will also feed into the design of the TAE's next-step machine, Copernicus, which will be an FRC device with NBI and electrode biasing. Specifically, the work may allow to identify machine configurations and physics operating states which have good stability properties. This would bring a very large mitigation of technical and financial risk.

List of presentations, publications; code development; models

1. International workshop on Open Magnetic Systems for Plasma confinement August 2021, Korea (virtual) (OS2021) conference. Sean Dettrick invited talk: "Recent Progress in Simulation and Theory of the C-2W Experiment".
2. TAE seminar July 22, 2021, E. Belova "Effects of NBI on FRC linear and nonlinear stability"
3. Several new options / models have been developed and implemented in the HYM code during this study: a) added sources/sinks in the full-f version of the HYM allowing to study their effects on stability, as well as option of the numerically calculated steady-state equilibria; b) new option of treating both the thermal and beam ions kinetically with different distribution functions and allowing for a bulk plasma rotation (the implementation included a modified Grad-Shafranov solver, and full-f / delta-f schemes in 3D stability simulations).
4. An analytic theory describing stability of compressional type modes in FRC with large-orbit fast ions have been developed in cylindrical geometry for the first time.

5. BENEFITS TO THE FUNDING DOE OFFICE'S MISSION

The project contributes to broader understanding of the effects of NBI fast ions on stability of high beta systems, and development of numerical codes suitable for nonlinear studies of systems with large orbit beam ions. The project also strengthens the collaboration between TAE and PPPL, benefiting both parties by cross-fertilization of ideas and codes benchmarking.